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An Adjustment Function for Dealing with Similarities

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Abstract. The paper investigates gradual semantics that are able to deal with similarity between arguments. Following the approach that defines semantics with evaluation methods, i.e., a couple of aggregation functions, the paper argues for the need of a novel function, called *adjustment function*. The latter is responsible for taking into account similarity when it is available. It aims at reducing the strengths of attackers according to the possible similarities between them. The reason is that similarity is seen as redundancy that should be avoided, otherwise a semantics may return inaccurate evaluations of arguments. The paper proposes a novel adjustment function that is based on the well-known weighted h-Categorizer, and investigates its formal properties.

Keywords. Argumentation, Similarity, Adjustment Function, Gradual Semantics.

1. Introduction

Argumentation is a reasoning approach, which justifies claims by *arguments*. It starts by generating arguments and their links (forming an argumentation graph), then evaluates the arguments by so-called *semantics*, and finally identifies winning claims. An argumentation graph can be enriched with various additional information like weights on arguments, which can represent votes [1] or certainty degrees [2], weights on links between arguments, which can represent relevance [3,4] or again votes of users [5]. A similarity measure assessing how alike are pairs of arguments may also be provided [6,7].

Existence of similarity between arguments is inevitable in practice, as arguments generally share information. Hence, developing semantics that are able to take into account similarity is crucial for discarding any redundancy that may lead to inaccurate evaluation of arguments. This is particularly the case for gradual semantics and more precisely those that satisfy the *Counting* principle from [8], which states that every alive attacker affects its target. Consequently, the authors in [9] proposed some reasonable properties on how a gradual semantics should deal with similarity. Furthermore, they proposed three gradual semantics that deal with similarity. They all extend *h*-Categorizer [10] but differ in the way they remove redundancy that is due to similarity. However, the three approaches suffer from weaknesses as described in the related work section.

In this paper, we start first by extending the general framework for gradual semantics that was proposed in [11]. That framework defines a gradual semantics by an evaluation

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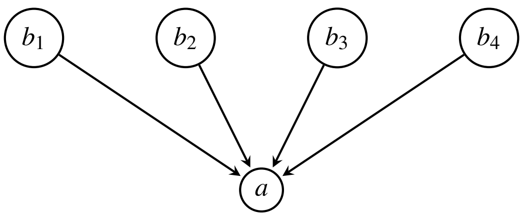
method, which is a tuple of three aggregation functions. This approach makes clear the different operations done by a semantics. We start by relaxing the strong constraint that all arguments are dissimilar. Indeed, we assume availability of similarities between arguments. Then, we extend the definition of evaluation method by introducing a fourth function, called adjustment function. It is responsible for reducing the strengths of attackers of an argument according to the similarity between them. Let us consider the following example of argumentation graph on the reduction of carbon emissions. Four arguments (b_1, b_2, b_3, b_4) are given and they all attack an argument (a):

- b_1: decreasing the population implies lower carbon emissions,
- b_2: reducing the use of aircraft implies lower carbon emissions,
- b_3: reducing distant imports implies lower carbon emissions,
- b_4: increasing local trade implies lower carbon emissions.

We can graphically represent this debate as follows:



In this example we can observe that there are some similarities between the arguments b_i. The more similar ones are b_3 and b_4 as they are about the same idea with dual writing. The argument b_2 has some similarity with b_3 and b_4 because reducing distant imports implies reducing the use of aircraft but not only (freighters, trains, trucks). And for the argument b_1 even if indirectly population touch everything, we can assume that this premise based on demography is different from the others (talking about transport or economy). Note that it is important to avoid these redundancies when evaluating the argument a. For that purpose, a reasonable semantics would start by evaluating the strength of each of the attackers b_i, then readjust those values by taking into account similarity. For instance, if a semantics assigns the value 1 to both b_3 and b_4 because they are not attacked, at the second step it may for instance decide to keep the whole strength of b_3 and set the value of b_4 to 0 due to the full similarity between the two arguments.

Another contribution of the paper consists of proposing a novel readjustment function. The latter distributes the burden of redundancy among attackers. In the previous example, the new function will decrease the value of both b_3, b_4. Furthermore, the function is based on the well-known weighted h-Categorizer that was proposed in the literature for a completely different purpose. Indeed, it is used as a gradual semantics for evaluating arguments. We investigate the properties of the novel function and compare it with the existing ones.

The paper is organised as follows: Section 2 introduces the argumentation framework, we are interested and extend the framework of gradual semantics proposed in [11]. Section 3 presents the novel readjustment function, and Section 4 investigates its properties. Section 5 is devoted to related work and the last section concludes.

2. Background

Throughout the paper, we denote by $\mathscr{U}$ the universe of all possible arguments, and consider argumentation frameworks as tuples made of a non-empty and finite subset of $\mathscr{U}$. Every argument has an initial weight that may represent different information (certainty degree of the argument's premises, credibility degree of its source, and so on). Arguments can attack each other and every attack is assigned a weight representing for instance its relevance as in case of analogical arguments [3]. We also assume availability of a similarity measure that assesses how alike are pairs of arguments.

For the sake of simplicity, all weights and similarities are elements of the unit interval [0,1]. The greater the value, the stronger the argument, or the more relevant an attack, or the more similar the pair of arguments.

Definition 1 (Similarity Measure) *A* similarity measure *on a set $X \subseteq_f \mathscr{U}$*[2] *is a function* $\mathbf{s} : X \times X \to [0,1]$ *such that:*

- $\forall a \in X$, $\mathbf{s}(a,a) = 1$,
- $\forall a,b \in X$, $\mathbf{s}(a,b) = \mathbf{s}(b,a)$.

The first condition states that every argument is fully similar to itself and the second states that similarity is a symmetric notion.

Definition 2 (AF) *An argumentation framework (AF) is a tuple* $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$, *where*

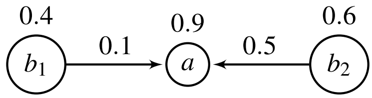
- $\mathscr{A} \subseteq_f \mathscr{U}$
- $\mathbf{w} : \mathscr{A} \to [0,1]$
- $\mathscr{R} \subseteq \mathscr{A} \times \mathscr{A}$
- $\sigma : \mathscr{R} \to [0,1]$
- $\mathbf{s} : \mathscr{A} \times \mathscr{A} \to [0,1]$

For $a,b \in \mathscr{A}$, $\mathbf{w}(a)$ denotes the initial weight of a, $\mathbf{s}(a,b)$ is the degree of similarity between a and b, $(a,b) \in \mathscr{R}$ means a attacks b, a is called *attacker* of b, $\sigma(a,b)$ is the degree of relevance of the attack, and $\mathtt{Att}(a)$ denotes the set of all attackers of a. Finally, the notation $\mathbf{s} \equiv 0$ denotes that there are no similarities between arguments.

In [12], the authors introduced for the first time *gradual semantics*, i.e, formal methods that evaluate strengths of arguments. Formally, they are functions that assign to every argument in an argumentation framework a value from an ordered scale. Examples of such semantics are h-Categorizer [10], Trust-based semantics [13], (DF)-Quad [14,15] and those proposed in [8].

In [11], the authors studied argumentation frameworks of the form $\langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \equiv 0 \rangle$, and have shown that a gradual semantics is defined using three functions. In order to better motivate those functions, let us consider the graph depicted below and focus on the argument a:

[2] $X \subseteq_f \mathscr{U}$ means X is a finite subset of $\mathscr{U}$.



In order to assess the strength of a, a gradual semantics proceeds in three steps:

1. To assess the *strength of every attack* (b_i, a). The idea is to combine the strength of the attacker b_i with the relevance degree of the attack $\sigma(b_i, a)$. This is done by a function $\mathbf{h}$. Since b_1, b_2 are not attacked, assume a semantics that keeps their initial weights, i.e., the strength of b_1 is 0.4 and the strength of b_2 is 0.6. Hence, $\alpha_1 = \mathbf{h}(0.4, \sigma(b_1, a)) = \mathbf{h}(0.4, 0.1)$ and $\alpha_2 = \mathbf{h}(0.6, \sigma(b_2, a)) = \mathbf{h}(0.6, 0.5)$, where α_i represents the strength of the attack (b_i, a).
2. To assess the *strength of the group of attacks* on a. This is done by an aggregation function $\mathbf{g}$, hence $\delta = \mathbf{g}(\alpha_1, \alpha_2)$.
3. To evaluate the impact of the two attacks on the initial weight of a. This is done by a function $\mathbf{f}$, hence $\lambda = \mathbf{f}(\mathbf{w}(a), \delta) = \mathbf{f}(0.9, \delta)$. This function returns the strengths of arguments, thus the strength of a is λ.

The tuple $\mathscr{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h} \rangle$ of the three functions is called in [11] an *evaluation method* of a gradual semantics. An important question is: how a semantics should consider similarities when they are available and at which step of the above process? As discussed in [9], a gradual semantics should take into account similarities between the attackers of an argument. In the above graph, assume that b_1 and b_2 are fully similar, i.e., $\mathbf{s}(b_1, b_2) = 1$. Note that b_1 is redundant wrt b_2, and thus considering both α_1 and α_2 will lead to an inaccurate evaluation of the argument a. Indeed, a will loose a lot of weight due to redundant information. Hence, before computing the strength of the group of attacks using the aggregation function $\mathbf{g}$, we introduce an *adjustment function* $\mathbf{n}$ that readjusts the two values considering the similarity between b_1 and b_2. This operation results in a *decrease* in the strength of the group of attacks. For instance, this function may keep the greatest value among α_1 and α_2 and sets the other to 0. Assume that $\alpha_1 > \alpha_2$, then $\mathbf{n}(\alpha_1, \alpha_2) = (\alpha_1, 0)$ and the strength of the group would be $g(\alpha_1, 0)$. Note that such function ignores the attack from b_2. In the next section, we provide a novel adjustment function that distributes the burden of redundancy among the two attackers. Before that, let us first extend the definition of evaluation method.

Definition 3 **(**`EM`**)** *An* evaluation method *(*`EM`*) is a tuple* $\mathscr{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{n} \rangle$ *such that:*

- $\mathbf{f} : [0,1] \times \mathtt{Range}(\mathbf{g})$ [3] $\rightarrow [0,1]$,
- $\mathbf{g} : \bigcup_{k=0}^{+\infty} [0,1]^k \rightarrow [0, +\infty[$,
- $\mathbf{h} : [0,1] \times [0,1] \rightarrow [0,1]$,
- $\mathbf{n} : \bigcup_{k=0}^{+\infty} \ ([0,1] \times \mathscr{U})^k \rightarrow [0,1]^k$.

Note also that the function $\mathbf{n}$ takes as input two kinds of input: k numerical values and k arguments. The reason is that the same values may not be adjusted in the same way depending on the similarity between the arguments to which they refer.

Let us now define formally a gradual semantics that deals with similarity.

[3] $\mathtt{Range}(\mathbf{g})$ denotes the co-domain of $\mathbf{g}$

Definition 4 (Gradual Semantics) *A gradual semantics $\mathscr{S}$ based on an evaluation method $\mathscr{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{n} \rangle$ is a function assigning to every* AF, $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$, *a weighting* $\mathtt{Deg}^{\mathscr{S}}_{\mathbf{G}} : \mathscr{A} \to [0,1]$ *such that for every* $a \in \mathscr{A}$, $\mathtt{Deg}^{\mathscr{S}}_{\mathbf{G}}(a) =$

$$\mathbf{f}\Bigg(\mathbf{w}(a), \mathbf{g}\Bigg(\mathbf{n}\Big((\mathbf{h}(\mathtt{Deg}^{\mathscr{S}}_{\mathbf{G}}(b_1), \sigma(b_1,a)), b_1), \cdots, (\mathbf{h}(\mathtt{Deg}^{\mathscr{S}}_{\mathbf{G}}(b_k), \sigma(b_k,a)), b_k)\Big)\Bigg)\Bigg),$$

where $\{b_1, \cdots, b_k\} = \mathtt{Att}(a)$. $\mathtt{Deg}^{\mathscr{S}}_{\mathbf{G}}(a)$ *is the* strength *of a.*

It has been shown in [11] that most of existing gradual semantics are instances of the above definition. An example is weighted h-Categorizer defined as follows:

Definition 5 (Weighted h-Categorizer Gradual Semantics) *Weighted h-categorizer semantics is a function* $\mathscr{S}_{\mathtt{wh}}$ *transforming any* AF, $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \equiv 0 \rangle$, *into a weighting* $\mathtt{Deg}^{\mathscr{S}_{\mathtt{wh}}}_{\mathbf{G}} : \mathscr{A} \to [0,1]$ *such that for every* $a \in \mathscr{A}$,

$$\mathtt{Deg}^{\mathscr{S}_{\mathtt{wh}}}_{\mathbf{G}}(a) = \begin{cases} \mathbf{w}(a) & \textit{iff } \mathtt{Att}(a) = \emptyset \\ \dfrac{\mathbf{w}(a)}{1 + \sum\limits_{b \in \mathtt{Att(a)}} \mathtt{Deg}^{\mathscr{S}_{\mathtt{wh}}}_{\mathbf{G}}(b) \times \sigma(b,a)} & \textit{else} \end{cases}$$

The above semantics uses an evaluation method $\mathscr{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h} \rangle$ such that:

$$\mathbf{f}_{\mathtt{frac}}(x_1, x_2) = \tfrac{x_1}{1+x_2} \;\Big\|\; \mathbf{g}_{\mathtt{sum}}(x_1, \cdots, x_n) = \textstyle\sum_{i=1}^{n} x_i \;\Big\|\; \mathbf{h}_{\mathtt{prod}}(x_1, x_2) = x_1 \times x_2$$

3. A Novel Adjustment Function

Throughout this section, we assume an arbitrary but fixed argumentation framework $\langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ and an arbitrary gradual semantics for evaluating its arguments. In what follows, we focus on the adjustment function of this semantics. We define this function, denoted by $\mathbf{n}_{\mathtt{wh}}$. The new function is nothing else than weighted h-Categorizer that is used in the literature as a gradual semantics for evaluating the strength of arguments. An important question is: why a gradual semantics can itself play the role of an adjustment function? The answer lies in the great analogy between the two: both aim at reducing strengths of arguments according to a set of other arguments. Another key question is: on which argumentation framework is the semantics applied? Recall that an input of any adjustment function is a tuple of the form $((x_1, b_1), \cdots, (x_n, b_n))$, with $x_i \in [0,1]$ is given by the gradual semantics that is used and $b_i \in \mathscr{A}$. For every such input, we create an argumentation framework $\langle \mathscr{A}', \mathbf{w}', \mathscr{R}', \sigma', \mathbf{s}' \rangle$ such that:

- $\mathscr{A}' = \{b_1, \cdots, b_n\}$
- For every $b_i \in \mathscr{A}'$, $\mathbf{w}'(b_i) = x_i$
- $\mathscr{R}' = (\mathscr{A}' \times \mathscr{A}') \setminus \{(b_i, b_i) \mid i = 1, \cdots, n\}$
- For every $(b_i, b_j) \in \mathscr{R}'$, $\sigma'((b_i, b_j)) = \mathbf{s}(b_i, b_j)$
- $\mathbf{s}' \equiv 0$

The framework contains thus the set of attackers whose strengths should be readjusted, the initial weight of every argument is its value assigned by the semantics, the attack relation is symmetric and the weight of every attack is the similarity degree between its target and its source. Weighted h-Categorizer is applied to this framework and the values assigned to arguments correspond to their readjusted values.

Definition 6 ($\mathbf{n}_{\mathtt{wh}}$) *Let* $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ *be an* AF, $x_1, \cdots, x_k \in [0,1]$, *and* $b_1, \cdots, b_k \in \mathscr{A}$. *We define the adjustment function* $\mathbf{n}_{\mathtt{wh}}$ *as follows:*

$$\mathbf{n}_{\mathtt{wh}}((x_1,b_1),\cdots,(x_k,b_k)) = (\mathtt{Deg}_{\mathbf{G}'}^{\mathscr{S}_{\mathtt{wh}}}(b_1),\cdots,\mathtt{Deg}_{\mathbf{G}'}^{\mathscr{S}_{\mathtt{wh}}}(b_k))$$

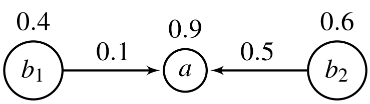
where $\mathbf{G}' = \langle \mathscr{A}', \mathbf{w}', \mathscr{R}', \sigma', \mathbf{s}' \rangle$, *such that:*

- $\mathscr{A}' = \{b_1, \cdots, b_k\}$,
- $\mathbf{w}'(b_1) = x_1, \cdots, \mathbf{w}'(b_k) = x_k$,
- $\mathscr{R}' = \{(b_1,b_2),\cdots,(b_1,b_k),\cdots,(b_k,b_1),\cdots,(b_k,b_{k-1})\}$,
- *For every* $(b_i,b_j) \in \mathscr{R}'$, $\sigma'((b_i,b_j)) = \mathbf{s}(b_i,b_j)$,
- $\mathbf{s}' \equiv 0$.

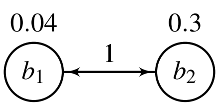
Hence, the strength x_i of every attacker b_i will be readjusted to $\mathtt{Deg}_{\mathbf{G}'}^{\mathscr{S}_{\mathtt{wh}}}(b_i)$, where

$$\mathtt{Deg}_{\mathbf{G}'}^{\mathscr{S}_{\mathtt{wh}}}(b_i) = \frac{x_i}{1 + \sum\limits_{j \in \{1,\cdots,n\} \setminus \{i\}} \mathtt{Deg}_{\mathbf{G}'}^{\mathscr{S}_{\mathtt{wh}}}(b_j) \times \mathbf{s}(b_j,b_i)}.$$

Example 1 *Let us illustrate the above definition on the graph below.*



Assume that b_1 *and* b_2 *are fully similar* ($\mathbf{s}(b_1,b_2) = 1$) *and let us consider a semantics that satisfies the Maximality principle from [8] according to which every non-attacked argument keeps its initial weight. Hence, the strength of* b_1 *is 0.4 and the strength of* b_2 *is 0.6. Assume also that to deal with the weight of relevance we use* $\mathbf{h}_{\mathtt{prod}}$ *then the adjustment function takes thus the tuples* $(0.04, b_1), (0.3, b_2)$ *as input. It builds the following argumentation framework:*



$\mathbf{n}_{\mathtt{wh}}$ *evaluates the arguments of the above graph using weighted h-Categorizer. It is easy to check that* $\mathtt{Deg}_{\mathbf{G}'}^{\mathscr{S}_{\mathtt{wh}}}(b_1) = 0.03$, $\mathtt{Deg}_{\mathbf{G}'}^{\mathscr{S}_{\mathtt{wh}}}(b_2) = 0.29$. *So,* $\mathbf{n}_{\mathtt{wh}}((0.04,b_1),(0.3,b_2)) = (0.03, 0.29)$ *meaning that the readjusted value of* b_1 *and* b_2 *are respectively* 0.03 *and* 0.29.

4. Properties

This section shows that the function $\mathbf{n}_{\mathtt{wh}}$ satisfies reasonable properties. The first result shows that $\mathbf{n}_{\mathtt{wh}}$ can be used by a gradual semantics. Namely, the gradual semantics that is based on the evaluation method $\langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{h}_{\mathtt{prod}}, \mathbf{n}_{\mathtt{wh}} \rangle$ assigns a unique strength to every argument.

Theorem 1 *There exists a unique semantics that is based on the evaluation method* $\langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{h}_{\mathtt{prod}}, \mathbf{n}_{\mathtt{wh}} \rangle$.

Proof In [11] the authors show that the weighted h-categorizer semantics can be defined by the evaluation method $\langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{h}_{\mathtt{prod}} \rangle$ and it has a unique semantics, i.e. it converge. More, the adjustment function $\mathbf{n}_{\mathtt{wh}}$ is the weighted h-categorizer semantics which modifies each weight of argument in its degree. Apply the adjustment function $\mathbf{n}_{\mathtt{wh}}$ before the aggregation function $\mathbf{g}_{\mathtt{sum}}$ changes the value of the arguments. This is equivalent to using $\langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{h}_{\mathtt{prod}} \rangle$ on a different graph. Therefore $\langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{h}_{\mathtt{prod}}, \mathbf{n}_{\mathtt{wh}} \rangle$ converge and has a unique semantics. ■

As expected from an adjustment function, the next property states that $\mathbf{n}_{\mathtt{wh}}$ can only reduce the value of an argument.

Proposition 1 *For any* AF, $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$, *for all* $a_1, \cdots, a_n \in \mathscr{A}$, *for all* $x_1, \cdots, x_n \in [0,1]$, *if* $\mathbf{n}_{\mathtt{wh}}((x_1,a_1), \cdots, (x_n,a_n)) = (x'_1, \cdots, x'_n)$, *then* $\forall i \in \{1, \cdots, n\}$, $x'_i \leq x_i$.

Proof Let $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ be an AF, $a_1, \cdots, a_n \in \mathscr{A}$ and $x_1, \cdots, x_n \in [0,1]$ such that $\mathbf{n}_{\mathtt{wh}}((x_1,a_1), \cdots, (x_n,a_n)) = (\mathtt{Deg}(a_1), \cdots, \mathtt{Deg}(a_n))$. For any $i \in \{1, \cdots, n\}$, from Definition 6, $\mathtt{Deg}(a_i) = \frac{x_i}{1+X}$ such that $X \in [0, +\infty[$ therefore $\mathtt{Deg}(a_i) \leq x_i$. ■

When all the arguments are dissimilar, the adjustment function does not alter the values of the arguments.

Proposition 2 *For any* AF, $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$, *for all* $a_1, \cdots, a_n \in \mathscr{A}$, *for all* $x_1, \cdots, x_n \in [0,1]$, *if* $\forall i, j \in \{1, \cdots, n\}$, $i \neq j$, $\mathbf{s}(a_i, a_j) = 0$, *then*

$$\mathbf{n}_{\mathtt{wh}}((x_1,a_1), \cdots, (x_n,a_n)) = (x_1, \cdots, x_n).$$

Proof Let $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ be an AF, $a_1, \cdots, a_n \in \mathscr{A}$ and $x_1, \cdots, x_n \in [0,1]$ such that $\forall i, j \in \{1, \cdots, n\}$, $i \neq j$, $\mathbf{s}(a_i, a_j) = 0$. From Definition 6, $\mathbf{n}_{\mathtt{wh}}((x_1,a_1), \cdots, (x_n,a_n)) = (\mathtt{Deg}(a_1), \cdots, \mathtt{Deg}(a_n))$ such that $\mathtt{Deg}(a_1) = \frac{x_1}{1+0}, \cdots, \mathtt{Deg}(a_n) = \frac{x_n}{1+0}$. ■

We show next that increasing the degree of similarity of a pair of arguments leads to the diminution of values of both arguments.

Proposition 3 *For any* AF, $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$, *for all* $a_1, a_2, b_1, b_2 \in \mathscr{A}$ *and for any* $x_1, x_2 \in \ ]0,1]$, *if*

- $\mathbf{n}_{\mathtt{wh}}((x_1,a_1), (x_2,a_2)) = (x'_1, x'_2)$,
- $\mathbf{n}_{\mathtt{wh}}((x_1,b_1), (x_2,b_2)) = (x''_1, x''_2)$,
- $\mathbf{s}(b_1, b_2) > \mathbf{s}(a_1, a_2)$,

then $x'_1 > x''_1$ *and* $x'_2 > x''_2$.

Proof Let $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ be an $\mathtt{AF}$, $a_1, a_2, b_1, b_2 \in \mathscr{A}$ and $x_1, x_2 \in]0,1]$ such that

- $\mathbf{n}_{\mathtt{wh}}((x_1,a_1),(x_2,a_2)) = (\mathtt{Deg}(a_1), \mathtt{Deg}(a_2))$,
- $\mathbf{s}(b_1,b_2) = \mathbf{s}(a_1,a_2) + \alpha$ such that $\alpha \in]0,1]$ and $\mathbf{n}_{\mathtt{wh}}((x_1,b_1),(x_2,b_2)) = (\mathtt{Deg}(b_1), \mathtt{Deg}(b_2))$.

From the definition 6,

$$\mathtt{Deg}(a_1) = \frac{x_1}{1 + \mathtt{Deg}(a_2) \times \mathbf{s}(a_1,a_2)} \qquad \mathtt{Deg}(a_2) = \frac{x_2}{1 + \mathtt{Deg}(a_1) \times \mathbf{s}(a_1,a_2)}$$

$$\mathtt{Deg}(b_1) = \frac{x_1}{1 + \mathtt{Deg}(b_2) \times (\mathbf{s}(a_1,a_2) + \alpha)} \qquad \mathtt{Deg}(b_2) = \frac{x_2}{1 + \mathtt{Deg}(b_1) \times (\mathbf{s}(a_1,a_2) + \alpha)}$$

Let us develop the equation of $\mathtt{Deg}(a_1)$:

$$\mathtt{Deg}(a_1) = \frac{x_1}{1 + \mathbf{s}(a_1,a_2) \times \frac{x_2}{1+\mathtt{Deg}(a_1)\times \mathbf{s}(a_1,a_2)}}$$

$$\Longleftrightarrow \mathtt{Deg}(a_1) = \frac{x_1}{\frac{1+\mathbf{s}(a_1,a_2)\times \mathtt{Deg}(a_1) + \mathbf{s}(a_1,a_2)\times x_2}{1+\mathbf{s}(a_1,a_2)\times \mathtt{Deg}(a_1)}}$$

$$\Longleftrightarrow \mathtt{Deg}(a_1) = \frac{x_1 + \mathbf{s}(a_1,a_2) \times (\mathtt{Deg}(a_1) \times x_1)}{1 + \mathbf{s}(a_1,a_2) \times (\mathtt{Deg}(a_1) + x_2)}$$

In a same way we can develop the equation of $\mathtt{Deg}(b_1)$:

$$\mathtt{Deg}(b_1) = \frac{x_1 + (\mathbf{s}(a_1,a_2) + \alpha) \times (\mathtt{Deg}(b_1) \times x_1)}{1 + (\mathbf{s}(a_1,a_2) + \alpha) \times (\mathtt{Deg}(b_1) + x_2)}.$$

Given that $x_1, x_2 \in]0,1]$ then $\alpha \times \mathtt{Deg}(b_1) \times x_1 < \alpha \times (\mathtt{Deg}(b_1) + x_2)$. Therefore $\mathtt{Deg}(b_1) < \mathtt{Deg}(a_1)$. We can do the same reasoning with a_2 and b_2 and we obtain that $\mathtt{Deg}(b_2) < \mathtt{Deg}(a_2)$. ■

When an argument is dissimilar to all other arguments, then we show that if its initial value is 0, then it will not have any impact on the readjusted values of the other arguments. This property is violated by one of the adjustment functions defined in [9] (see the related work section).

Proposition 4 *For any* $\mathtt{AF}$, $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$, *for all* $a_1, \cdots, a_n, b \in \mathscr{A}$, *for all* $x_1, \cdots, x_n, y \in [0,1]$, *if*

- $\forall i \in \{1, \cdots, n\}$, $\mathbf{s}(a_i, b) = 0$,
- $y = 0$,

then $\mathbf{n}_{\mathtt{wh}}((x_1,a_1), \cdots, (x_n,a_n), (y,b)) = (\mathbf{n}_{\mathtt{wh}}((x_1,a_1), \cdots, (x_n,a_n)), 0)$.

More strongly, we show that an argument having an initial value of 0 and for any similarity with other arguments, this arguments doesn't impact the readjusted values of the other arguments.

Proposition 5 *For any* AF, $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$, *for all* $a_1, \cdots, a_n, b \in \mathscr{A}$, *for all* $x_1, \cdots, x_n, y \in [0,1]$, *if*

- $y = 0$,

then $\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n), (y, b)) = (\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n)), 0)$.

Proof Let $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ be an AF, $a_1, \cdots, a_n, b_1 \in \mathscr{A}$ and $x_1, \cdots, x_n, y \in [0,1]$ such that

- $y = 0$.

From Definition 6 we have $\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n)) = (\mathtt{Deg_1}(a_1), \cdots, \mathtt{Deg_1}(a_n)) = \mathtt{Deg_1}^{\mathscr{S}_{\mathtt{wh}}}_{G'}$, where

$$\mathtt{Deg_1}^{\mathscr{S}_{\mathtt{wh}}}_{G'} = \begin{cases} \mathtt{Deg_1}(a_1) = \frac{x_1}{1+\mathtt{Deg_1}(a_2)\times\mathbf{s}(a_1,a_2)+\cdots+\mathtt{Deg_1}(a_n)\times\mathbf{s}(a_1,a_n)} \\ \cdots \\ \mathtt{Deg_1}(a_n) = \frac{x_n}{1+\mathtt{Deg_1}(a_1)\times\mathbf{s}(a_n,a_1)+\cdots+\mathtt{Deg_1}(a_{n-1})\times\mathbf{s}(a_n,a_{n-1})} \end{cases}$$

and $\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n), (y, b_1)) = (\mathtt{Deg_2}(a_1), \cdots, \mathtt{Deg_2}(a_n)) = \mathtt{Deg_2}^{\mathscr{S}_{\mathtt{wh}}}_{G'}$, where

$$\mathtt{Deg_2}^{\mathscr{S}_{\mathtt{wh}}}_{G'} = \begin{cases} \mathtt{Deg_2}(a_1) = \frac{x_1}{1+\mathtt{Deg_2}(a_2)\times\mathbf{s}(a_1,a_2)+\cdots+\mathtt{Deg_2}(a_n)\times\mathbf{s}(a_1,a_n)+\mathtt{Deg_2}(b_1)\times\mathbf{s}(a_1,b_1)} \\ \cdots \\ \mathtt{Deg_2}(a_n) = \frac{x_n}{1+\mathtt{Deg_2}(a_1)\times\mathbf{s}(a_n,a_1)+\cdots+\mathtt{Deg_2}(a_{n-1})\times\mathbf{s}(a_n,a_{n-1})+\mathtt{Deg_2}(b_1)\times\mathbf{s}(a_n,b_1)} \\ \mathtt{Deg_2}(b_1) = \frac{y_1}{1+\mathtt{Deg_2}(a_1)\times\mathbf{s}(b_1,a_1)+\cdots+\mathtt{Deg_2}(a_n)\times\mathbf{s}(b_1,a_n)} \end{cases}$$

Given that $y = 0$, $\mathtt{Deg_2}(b_1) = 0$, so for every $i \in \{1, \cdots, n\}$, $\mathtt{Deg_1}(a_i) = \mathtt{Deg_2}(a_i)$. ■

The function $\mathbf{n}_{\mathtt{wh}}$ cannot readjusts a positive value to 0. This means that it does not ignore any attacker when similarities are available. It rather distributes the burden of redundancy among attackers.

Proposition 6 *Let* $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ *be an* AF, $a_1, \cdots, a_n \in \mathscr{A}$, $x_1, \cdots, x_n \in [0,1]$ *and* $\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n)) = (x'_1, \cdots, x'_n)$. *For any* $i \in \{1, \cdots, n\}$, *if* $x_i > 0$, *then* $x'_i > 0$.

Proof Let $\mathbf{G} = \langle \mathscr{A}, \mathbf{w}, \mathscr{R}, \sigma, \mathbf{s} \rangle$ be an AF, $a_1, \cdots, a_n \in \mathscr{A}$, $x_1, \cdots, x_n \in [0,1]$ and $\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n)) = (\mathtt{Deg}(a_1), \cdots, \mathtt{Deg}(a_n))$. For any $i \in \{1, \cdots, n\}$, from Definition 6, $\mathtt{Deg}(a_i) = \frac{x_i}{1+X}$ such that $X \in [0, +\infty[$ therefore if $x_i > 0$, then $\mathtt{Deg}(a_i) > 0$. ■

5. Related Work

In [9], the authors proposed three gradual semantics dealing with similarity in argumentation frameworks that are free of weights on attacks. We are interested in comparing the adjustment functions, hence the lack of function **h** is not a problem. However, from the three gradual semantics, one of them (Grouping weighted h-categorizer - `GHbs`) has not an independent adjustment function, that means this gradual semantics mixed the aggregation function (**g**) with the adjustment function (**n**). That is why we will not compare this method with our own.

The first semantics that we will compare is the Extended weighted h-Categorizer (`EHbs`) which uses a similarity measure between set of arguments. The principle of its adjustment function used consists in two steps:

1. ordering arguments from the strongest to the weakest ones (depending on their strengths),
2. then after permutation, from each argument the function keeps only the proportion of novelty according to the previous arguments already adjusted.

As described in the background section, there exist different strategies to distribute the similarity. In this function, the similarity is applied on sub-set of arguments according to its rank in the permutation. The first argument of this permutation will for instance keep all its initial weight. Another strategy can be to distribute the diminution on both arguments as done by our $\mathbf{n}_{\mathtt{wh}}$ function (proposition 3). These different strategies of adjustment are relevant for some aggregation functions **g** and not for others. For instance, when **g** is the aggregation function $\mathbf{g}_{\mathtt{max}}$, i.e. returning the maximal value of a set; distributing redundancy will make a significant difference in the evaluation.

Moreover, it can be noted that the way to ordering the attackers is not determinative, i.e. the constraint producing the ranking (only by degree) is not always unique (there may be ties) and these different rankings may produce different adjustments. For instance, if 3 arguments a, b, c have the same degree x but not the same similarity between them then the ordering will change the adjustment.

The second semantics that we will compare is the Readjustment weighted h-Categorizer (`RHbs`) which uses a binary similarity measure like our semantics. Let us introduce its adjustment function named Readjusted score. This function is based on different averages. We can describe its process in two operations to adjust the degree of an argument a:

1. for each other arguments x, it compute an average adjusted score α between x and a,
2. the final adjusted degree of a is the average of all the average adjusted score α.

We denote by `avg` the average operator. Formally the definition is the following:

Definition 7 ($\mathbf{n}_{\mathtt{rs}}$) *Let* $a_1, \cdots, a_k \in \mathscr{U}$ *and* $x_1, \cdots, x_k \in [0,1]$. $\mathbf{n}_{\mathtt{rs}}((x_1,a_1), \cdots, (x_k,a_k)) =$

$$\left(\underset{x_i \in \{x_1,\cdots,x_k\} \setminus \{x_1\}}{\mathtt{avg}} \left(\frac{\mathtt{avg}(x_1,x_i) \times (2-\mathbf{s}(a_1,a_i))}{2} \right), \cdots, \right.$$

$$\left. \underset{x_i \in \{x_1,\cdots,x_k\} \setminus \{x_k\}}{\mathtt{avg}} \left(\frac{\mathtt{avg}(x_k,x_i) \times (2-\mathbf{s}(a_k,a_i))}{2} \right) \right).$$

$\mathbf{n}_{\mathtt{rs}}() = ()$ *and* $\mathbf{n}_{\mathtt{rs}}((x_1,a_1)) = (x_1)$ *if* $k = 1$.

To compare $\mathbf{n}_{\mathtt{wh}}$ with $\mathbf{n}_{\mathtt{rs}}$ let's come back to the example 1.

Example 1 (Cont) As reminder, $x_1 = 0.04$, $x_2 = 0.3$ and $\mathbf{s}(b_1, b_2) = 1$.
Then $\mathbf{n}_{\mathtt{rs}}((x_1, b_1), (x_2, b_2)) = (0.085, 0.085)$ while $\mathbf{n}_{\mathtt{wh}}((x_1, b_1), (x_2, b_2)) = (0.03, 0.29)$.

Moreover, we propose the new adjustment function $\mathbf{n}_{\mathtt{wh}}$, because the Readjusted score violate some intuitive proposition.

Proposition 7 *The adjustment function $\mathbf{n}_{\mathtt{rs}}$ violates proposition 2, i.e. it does alter the values of the arguments when all the arguments are dissimilar.*

Proof Let $a, b \in \mathscr{U}$ such that $\mathbf{s}(a, b) = 0$ and $x_a = 1$, $x_b = 0.8$, then $\mathbf{n}_{\mathtt{rs}}((x_a, a), (x_b, b)) = (0.9, 0.9)$. ■

Proposition 8 *The adjustment function $\mathbf{n}_{\mathtt{rs}}$ violates proposition 4, i.e. an argument dissimilar to all other and whose its initial value is 0, can have an impact on the readjusted values of the other arguments.*
In addition, using the aggregation function $\mathbf{g}_{sum}$ there exist $a_1, \cdots, a_n, b \in \mathscr{U}$ and $x_1, \cdots, x_n, y \in [0, 1]$ such that:

- $\forall i \in \{1, \cdots, n\}$, $\mathbf{s}(a_i, b) = 0$,
- $y = 0$,
- $\mathbf{g}_{sum}(\mathbf{n}_{\mathtt{rs}}((x_1, a_1), \cdots, (x_n, a_n))) < \mathbf{g}_{sum}(\mathbf{n}_{\mathtt{rs}}((x_1, a_1), \cdots, (x_n, a_n), (y, b)))$.

This means that adding an attacker dissimilar to all other and whose its initial value is 0, can increase the sum of readjusted values of the set of attackers.

Proof Let $a, b, c \in \mathscr{U}$ such that $\mathbf{s}(a, b) = 0.5$, $\mathbf{s}(a, c) = 0$, $\mathbf{s}(b, c) = 0$ and $x_a = 1$, $x_b = 0.8$, $x_c = 0$ then $\mathbf{n}_{\mathtt{rs}}((x_a, a), (x_b, b)) = (0.675, 0.675)$ and $\mathbf{n}_{\mathtt{rs}}((x_a, a), (x_b, b), (x_c, c)) = (0.5875, 0.5375, 0.45)$. Moreover, we have that $0.675 + 0.675 = 1.35 < 1.575 = 0.5875 + 0.5375 + 0.45$. ■

6. Conclusion

The paper extended the general framework for gradual semantics proposed in [11]. The latter defines a gradual semantics with evaluation methods, which are tuples of three aggregation functions. In this paper, we relaxed the constraint that arguments are all dissimilar. We assumed thus the existence of a similarity measure on the set of arguments. We extended the definition of evaluation method by introducing a novel adjustment function. The latter is responsible for taking into account similarity. We also proposed an instance of such function, which is based on the weighted h-Categorizer. Note that the latter is used in the literature for a completely different reason, namely as a gradual semantics. We investigated the properties of the function, and have shown that it can safely be used by a semantics including h-Categorizer itself. This would mean that h-Categorizer can be used as an adjustment function of a semantics and as the semantics itself.

This work can be extended in different directions. One of them is to study adjustment functions more generally in evaluation methods. The objective would be to give the crucial properties of a reasonable adjustment function.

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